

# Fully Robust Tree-Diffie-Hellman Group Key Exchange

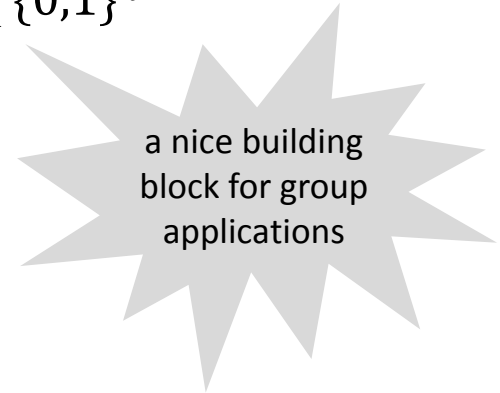
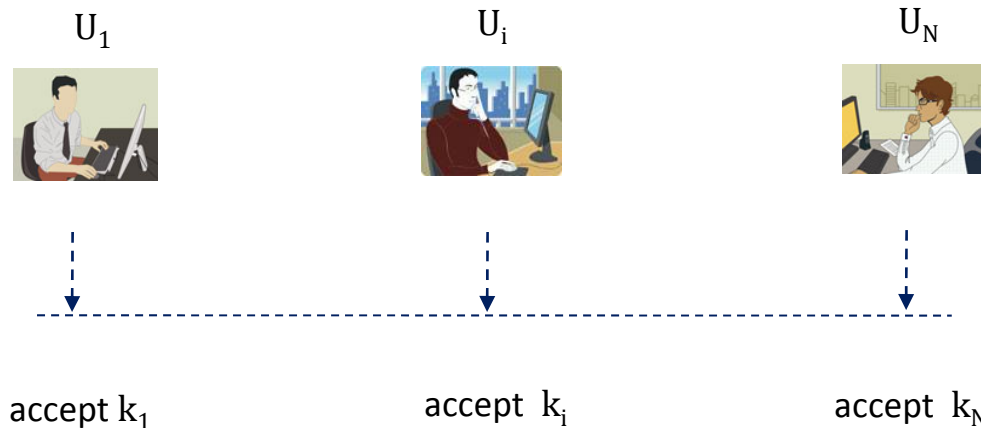
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# Group Key Exchange

Users in  $U = \{U_1, \dots, U_N\}$  run a Group Key Exchange (GKE) Protocol and compute a session group key  $k$  indistinguishable from  $k^* \in_R \{0,1\}^k$



secure (private and authenticated) group channel for  $U_1, \dots, U_N$

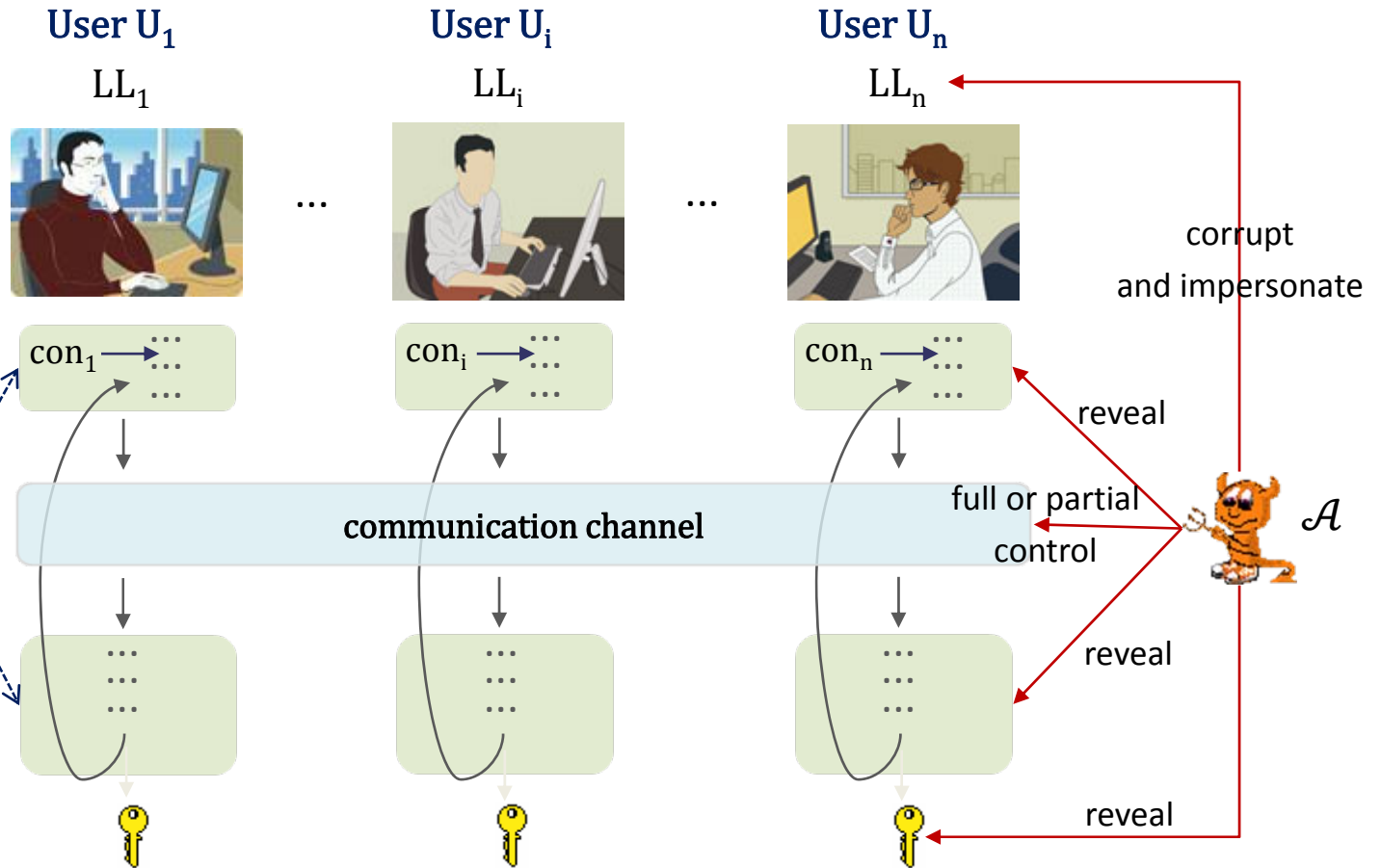
# Adversary

long-lived keys  
for authentication  
( $sk_i, pk_i$ ) or  $pw_i$

ephemeral secrets  
for key derivation  
kept in internal states

$state_i$

session group key



# Diverse Threats and Requirements

## Outsider Security

[BCPQ01,KY03,BMS07,BM08,GBG09]

indistinguishability of session keys  
authentication/impersonation attacks  
forward secrecy  
key-compromise impersonation  
⇒ **AKE-Security**

there are compilers  
authentication compilers  
[KY03,BMS07]

## Insider Security (optional)

[KS05,DPSW06,BM08,GBG09]

mutual authentication  
key confirmation  
key compromise impersonation  
⇒ **MA-Security**

key replication, control  
contributiveness  
⇒ **Contributiveness**

there are compilers  
for MA-security and  
contributiveness  
[KS05,BM07,BM08]

## Robustness

[CS04,DPSW06,KT07,KT08]

in general the goal of robustness is to ensure fault-tolerance  
(users should be able to proceed and compute session keys despite of identified failures)

# Non-Robust GKE

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## Problems in non-robust GKE protocols

outsider and optionally insider attacks are prevented

**but at the cost that the protocol execution aborts!**

## Robust GKE

protocol execution should continue despite of

*network faults*

sent messages are not delivered properly

*system crashes*

the system remains inoperable and needs restart

*malicious user behavior*

essentially the *insider attacks*

## Fully Robust GKE<sup>[JKT07]</sup>

protocol execution succeeds despite of up to  $n-2$  failed users

# Amir et al.'s Robust GKE

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## GCS-based solution<sup>[AN-RSSKT01]</sup>

execute *dynamic GKE protocol* (e.g. [S02])  
(handles joins and leaves of users)

on top of a *membership view-based group communication system*  
ensures reliable communication  
updates the set of alive users in a consistent way

## Observations

execution of GKE can still abort

GKE protocol has to be restarted if GCS notices a failure

# Cachin-Strobl's Robust GKE

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Consensus-based solution<sup>[CS04]</sup>

*asynchronous reliable channel with authentication*

generalized abstraction of Burmester-Desmedt protocol<sup>[BD94]</sup>

fault-tolerance via *k-resilient consensus protocol*<sup>[CKS00,CR01]</sup>

achieves strong AKE-security

for the optimal bound of  $n - 2k$  corrupted users

## Observations

does not address insider attacks

not fully robust ... as a consequence of the consensus protocol

(the optimal bound holds only for the asynchronous communication)

# Desmedt et al.'s Robust GKE

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VSS-based solution<sup>[DPSW06]</sup>

*weakly synchronized reliable broadcast channel without authentication*

fault tolerance via *(k-out-of-n) VSS technique*<sup>[P91]</sup>

modified Katz-Yung technique<sup>[KY03]</sup> for authentication

achieves weak AKE / MA and non-malleability

non-malleability is stronger than contributiveness

*but* the corruption model is weak

## Observations

not fully-robust ... as a consequence of VSS technique

assumes weak corruption model



# Jarecki et al.'s Robust GKE

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„Transitive closure of a circle“-based solution<sup>[JKT07]</sup>

*weakly synchronized reliable broadcast channel with authentication*

4 protocols that differ in complexity / robustness

fault tolerance via *a new circle-replication technique* for [BD94]

multiple circles with different subsets of users

## Observations

does not consider active/impersonation attacks

does not consider insider attacks

assumes that each user fails with the same probability

# In Comparison to this...

GKE	Out- /Insider Security				Robustness	Costs		
	AKE	MA	Con	Model	max. #Faults	Rnd	BCast	Ops/ User
VSS-based <sup>[DPSW06]</sup>	w	w	w	STD	$n/2 - 1$	8	$O(nk)$	$O(n)$
BD-RGKA <sup>[JKT07]</sup>	w	-	-	STD	$n - 2$	3	$O(n^3)$	$O(n^2)$
RGKA <sup>[JKT07]</sup>	w	-	-	STD	$n - 2$	3	$O(n^2)$	$O(n)$
t-RGKA <sup>[JKT07]</sup>	w	-	-	STD	$2t - 1$	3	$O(nt)$	$O(t)$
RGKA <sup>[JKT07]</sup>	w	-	-	STD	$n - 2$	$O(\delta)$	$O(n \log n)$	$O(n)$
R-TDH1	s	-	-	STD	$n - 2$	3	$O(n^2)$	$O(n)$
IR-TDH1	s	s	s	ROM	$n - 2$	3	$O(n^2 l)$	$O(nl)$
TDH1 <sup>[BM08]</sup>	s	s	s	STD	0	3	$O(n)$	$O(n)$

w – weak corruptions (reveal LLs)

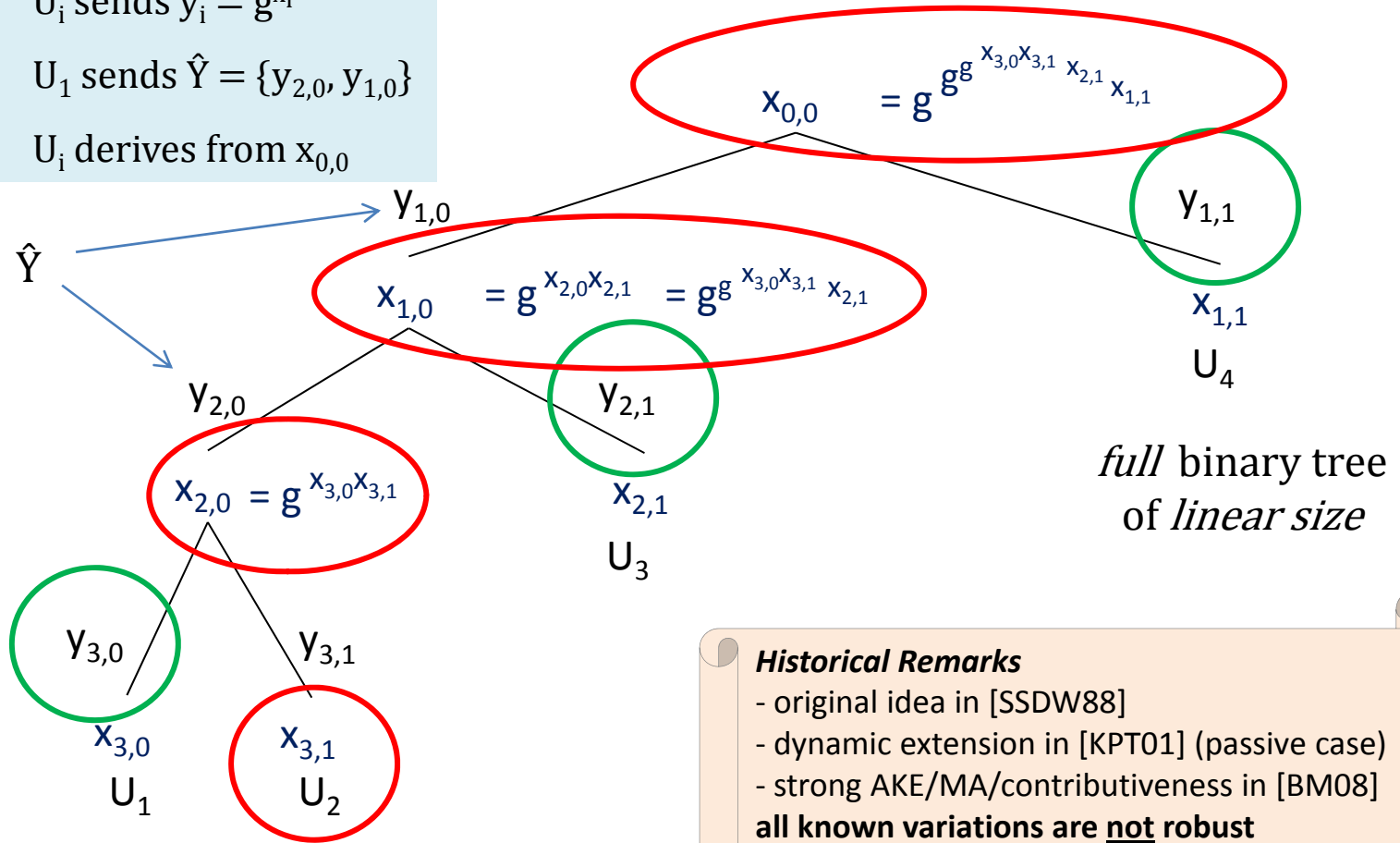
s – strong corruptions (reveal LLs and states)

# Tree Diffie-Hellman (simplified)

Round 1  $U_i$  sends  $y_i = g^{x_i}$

Round 2  $U_1$  sends  $\hat{Y} = \{y_{2,0}, y_{1,0}\}$

Key  $U_i$  derives from  $x_{0,0}$



# Communication Channel and Model

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## Channel

*weakly synchronized reliable broadcast without authentication* i.e. [DPSW06,JKT07]

## Broadcast of Round Messages

for each round  $\mathcal{A}$  is given the set  $S$  of round messages (of honest users)

the *round timer* is started (sufficiently large to cover delays)

$\mathcal{A}$  can modify the set  $S$  (e.g., change/inject, order/delete messages)

$\mathcal{A}$  outputs modified set  $S'$  (prior to timer expiration)

## Delivery of Round Messages

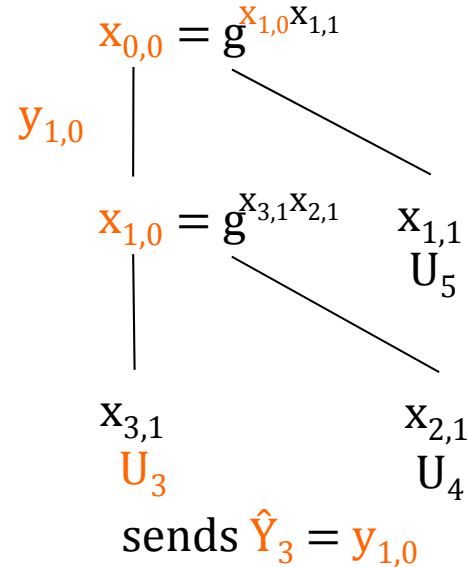
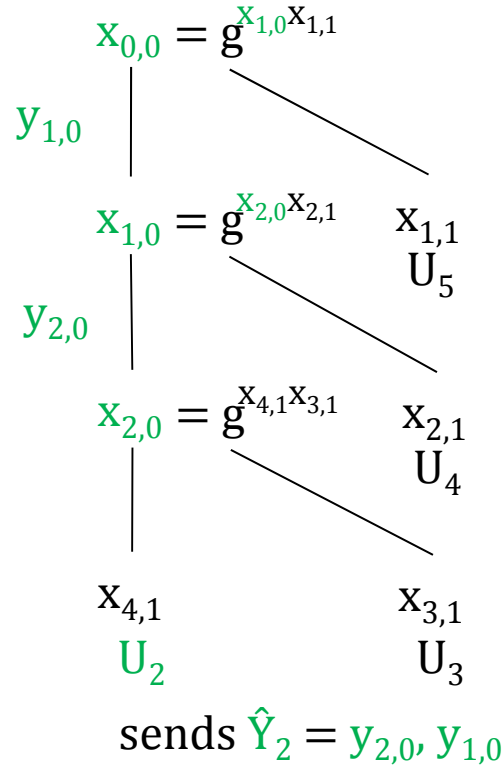
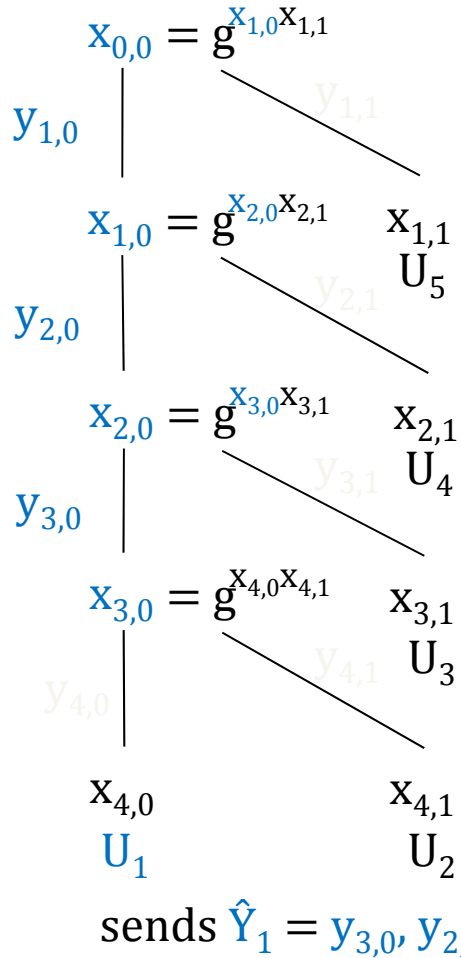
$S'$  determines which users are still active/alive/connected

messages in  $S'$  are delivered to all connected users

each connected user updates its own *set of active* users (denoted  $\text{pid}$ )

# „Tree-Replication Technique“

(applies in Round 2)



$U_4$  and  $U_5$   
send 'alive'

choose  $\hat{Y}_\gamma$  from all round messages

$\gamma$  : lowest-index of a user  $U_\gamma$

compute  $x_{0,0}$  using  $\hat{Y}_\gamma$

if  $\gamma = 4$  then  $x_{0,0} = g^{x_{2,1}x_{1,1}}$

# R-TDH1 (Outsider Security) I

## Preliminaries

$\text{pid}_i = U_1 | \dots | U_n$

public constant  $v_0$

$U_1$

$U_2$

$U_3$

$U_4$

$U_5$

## Round 1 (Broadcast)

$U_i$  broadcasts  $U_i | 1 | r_i$

$r_1$

$r_2$

$r_3$

$r_4$

$r_5$

## Round 2 (Broadcast)

$U_i$  updates  $\text{pid}_i$  and aborts\* if  $\text{pid}_i = U_i$

$\text{nonces}_i = r_1 | \dots | r_n$

broadcasts  $U_i | 2 | y_i | \sigma'_i$

$y_1 = g^{x_1}$

$y_2 = g^{x_2}$

$y_3 = g^{x_3}$

$y_4 = g^{x_4}$

$y_5 = g^{x_5}$

$\sigma'_1$

$\sigma'_2$

$\sigma'_3$

$\sigma'_4$

$\sigma'_5$

\* abort implies erasure of internal states

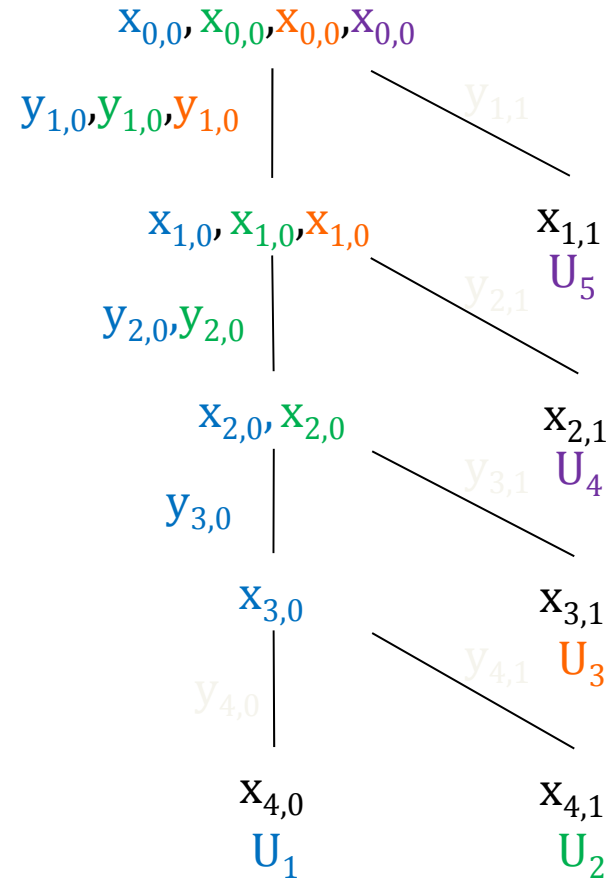
# R-TDH1 (Outsider Security) II

## Round 3 (Broadcast)

$U_i$  updates  $\text{pid}_i$  and nonces  $\sigma_i$   
 computes  $X_i$  (incl.  $x_{0,0}$ ) and  $\hat{Y}_i$   
 broadcasts  $U_i | 3 | \hat{Y}_i | \sigma_i$   
 $U_4, U_5$  broadcast  $U_{4|5} | 3 | \text{'alive'} | \sigma_i$

## Group key derivation

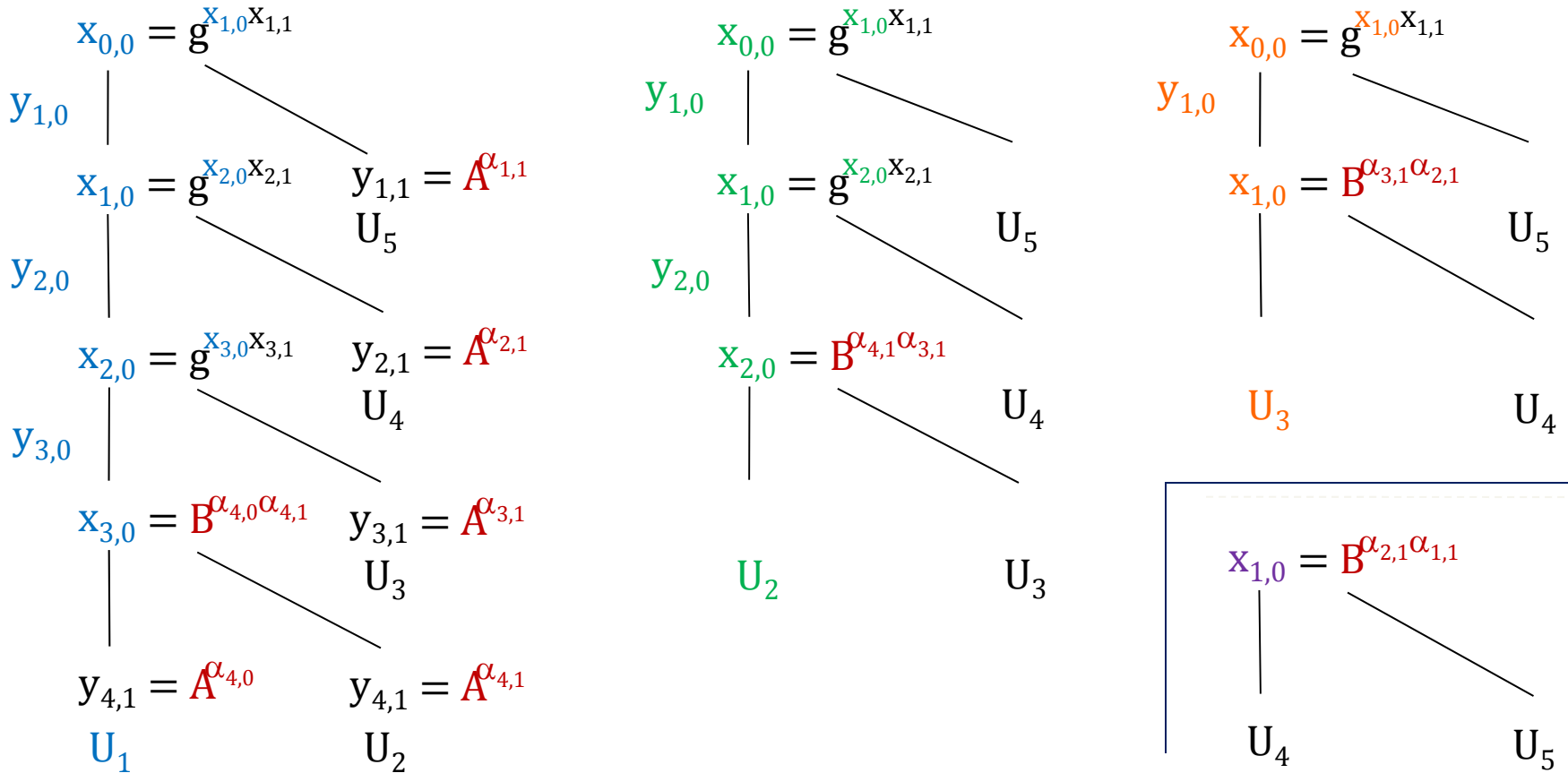
$U_i$  updates  $\text{pid}_i$  and nonces  $\sigma_i$   
 determines  $\gamma$   
 computes  $x_{0,0}$  using  $\hat{Y}_\gamma$   
 computes  $K_i = \text{PRF}_{x_{0,0}}(v_0)$   
 erases all ephemeral secrets  
 accepts  $K_i$



# Security of „Tree Replication Technique“

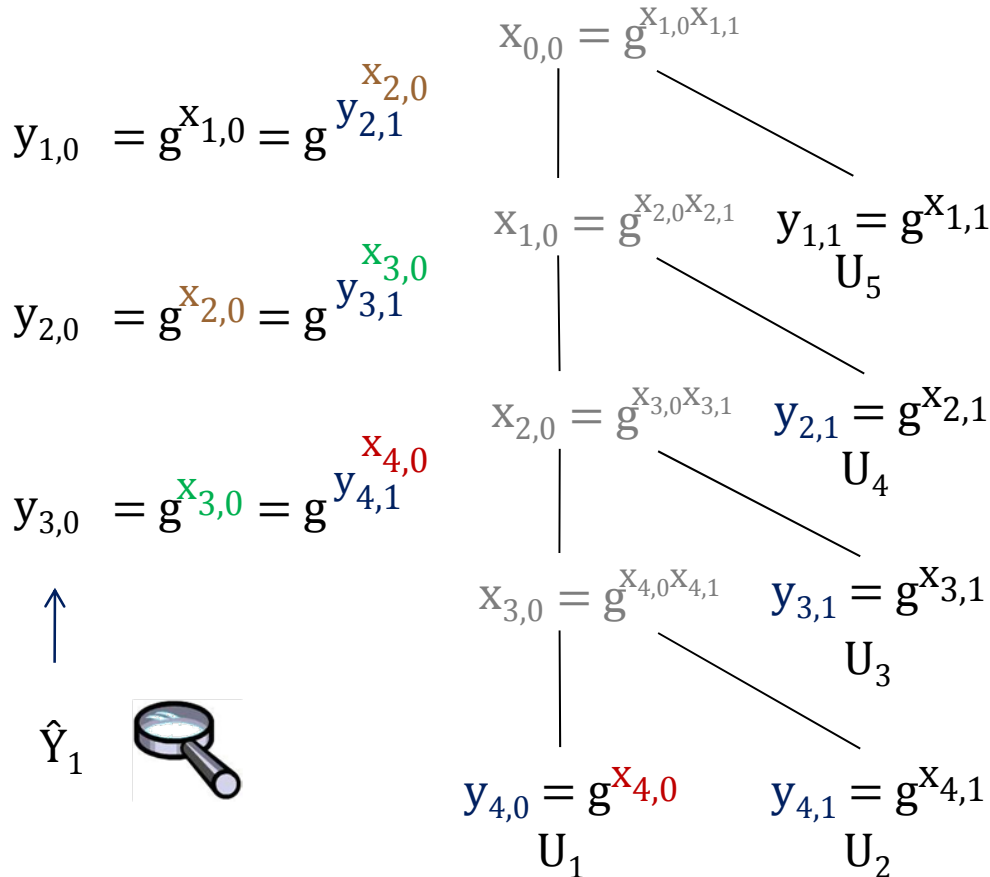
Square-Exponent Decisional Diffie-Hellman (SEDDH) Assumption [W99,SS01]

given  $(g, A=g^a, B)$  decide whether  $B=g^{a^2}$  or  $g^b$





# Consistency of $\hat{Y}_i$ for Insider Security



$U_i$  proves in ZK [S96, AST02]

$$\text{Log}_g(y_{l,0}) = \text{Log}_{y_{l,1}}(\text{Log}_g(y_{l-1,0}))$$

starting with own  $y_i = g^{x_i}$

$$y_{2,0} = g^{x_{2,0}} \quad \wedge \quad y_{1,0} = g^{y_{2,1}^{x_{2,0}}}$$

$$y_{3,0} = g^{x_{3,0}} \quad \wedge \quad y_{2,0} = g^{y_{3,1}^{x_{3,0}}}$$

$$y_{4,0} = g^{x_{4,0}} \quad \wedge \quad y_{3,0} = g^{y_{4,1}^{x_{4,0}}}$$

# Conclusion

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## R-TDH1: Optimally Robust TDH1 with Outsider Security

robustness of Tree-Diffie-Hellman<sup>[SSDW88,KPT01,BM08]</sup> via „Tree Replication Technique“  
authentication via signatures<sup>[KY03,DPSW06]</sup>  
outsider security (AKE-security) in the standard model

## IR-TDH1: Optimally Robust THD1 with Insider Security

consistency of tree computations via NIZK proofs  
key confirmation via signatures<sup>[KS05,BMS07]</sup>  
insider security (MA-security/contributiveness) in the random oracle model

## Unification of Security Models (not covered by the talk)

stronger security definitions from <sup>[BM08,GBG09]</sup>  
strong AKE-/MA-security, contributiveness  
robustness from <sup>[DPSW06]</sup>, optimality from <sup>[JKT07]</sup>  
non-authenticated reliable broadcast with weak synchrony

